When $m = n + 1/2$, n as above, we have

$$f(x) = k\,\frac{1\cdot 3\cdot 5\cdot\ \cdots\ (2n+1)}{2^n(n-1)\,!}\,x^{n-1}. \tag{6}$$

When $n = 2$, (5) gives the parabola, in which the quantity of flow is proportional to the square of the depth of stream, and (6) gives the triangular weir notch, with flow proportional to $h^{5/2}$.

It is easy now to show that (4) is still a solution of (1), though not necessarily continuous, for values of $m > 1/2$. To do this we put $x = ht$ in (1), which gives

$$\int_0^1 \sqrt{1-t}\, f(ht)dt = kh^{m-\frac{3}{2}}.$$

Substituting $f(ht) = ch^nt^n$ gives

$$c\int_0^1 \sqrt{1-t}\, t^n dt = kh^{m-n-\frac{3}{2}}.$$

The left member of this equation is independent of h, hence we must have

$$n = m - 3/2, \qquad \text{and} \qquad c\int_0^1 \sqrt{1-t}\, t^n dt = k,$$

which gives, since $\Gamma(3/2) = \sqrt{\pi}/2$,

$$c = \frac{2k}{\sqrt{\pi}}\cdot\frac{\Gamma(m+1)}{\Gamma(m-\frac{1}{2})}, \quad \text{provided } m > 1/2.$$

Hence (4) is a solution if $m > 1/2$.

When $m = 3/2$ we get the rectangular notch, $y =$ constant, and when $m = 1$ we get the curve $y = 1/\sqrt{x}$, such that the flow is directly proportional to the depth of stream.

DEPRECIATION BY A CONSTANT PERCENTAGE PLUS A CONSTANT.

By C. R. FORSYTH, Dartmouth College.

There are two ways in which a piece of property may depreciate which are usually considered in any complete treatise on the mathematical theory of depreciation, treatments in which possible interest accumulations are given no consideration. The methods employed to compute the annual or periodic allowance for depreciation corresponding to these two ways are known familiarly as the "straight line" method and the "constant percentage of book value" method. The annual allowance corresponding to the first case is the constant

$$k = \frac{C - S}{n}, \tag{1}$$

where C denotes the original cost, S the scrap value and n the estimated lifetime

of the property. The constant percentage in the second case is

$$x = 1 - \sqrt[n]{\frac{S}{C}}. \tag{2}$$

It is not the purpose of this paper to dwell upon the practical virtues of the application of the first method or the practical defects of the application of the second method but rather, since both methods occupy a fairly important place in the mathematical theory of depreciation from the point of view of the mathematical theory itself, to present results analogous to those given above when a piece of property depreciates by a constant percentage plus a constant.

If we denote the original cost by C, the constant percentage by x and the constant by k, then the amount to be charged off the first year is $Cx + k$ leaving a residue of $C(1 - x) - k$. Continuing this reasoning it is easily shown that the residue R_n at the end of the nth year will be

$$R_n = (1 - x)^n\left(C + \frac{k}{x}\right) - \frac{k}{x}. \tag{3}$$

Suppose that the valuation engineer is able to give at least three estimates R_n, R_{2n}, R_{3n}, etc., of the value of the property corresponding to the ends of at least three intervals of n years. Assuming first R_n to be known, equation (3) is easily solved for k to give

$$k = x\frac{R_n - C(1 - x)^n}{(1 - x)^n - 1}. \tag{4}$$

It remains then to determine x. Assuming now that R_{2n} and R_{3n} are also known, we may write

$$R_n = (1-x)^n\left(C + \frac{k}{x}\right) - \frac{k}{x}, \qquad R_{2n} = (1-x)^{2n}\left(C + \frac{k}{x}\right) - \frac{k}{x},$$
$$R_{3n} = (1-x)^{3n}\left(C + \frac{k}{x}\right) - \frac{k}{x}.$$

Subtracting and dividing as indicated on the left side of the following equation we obtain

$$\frac{R_{2n} - R_{3n}}{R_n - R_{2n}} = (1 - x)^n,$$

whence

$$x = 1 - \sqrt[n]{\frac{R_{2n} - R_{3n}}{R_n - R_{2n}}}. \tag{5}$$

As a simple illustration, suppose that a property depreciates annually (that is, $n = 1$) by a constant percentage plus a constant in accordance with the following data: $C = \$1000.00$, $R_1 = \$880.00$, $R_2 = \$772.00$, $R_3 = \$674.80$. Then, by formula (5)

$$x = 1 - \frac{772.00 - 674.80}{880.00 - 772.00} = 1/10.$$

By formula (4)

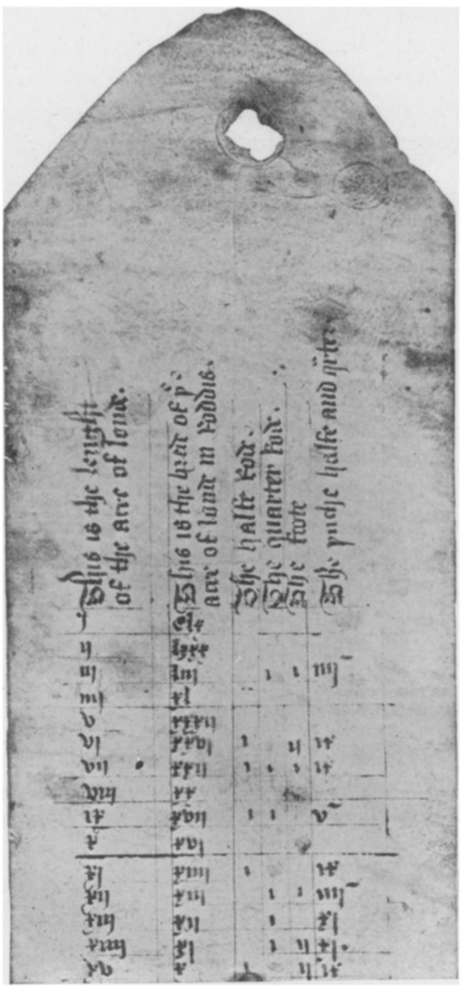
$$k = \frac{1}{10}\frac{\$880.00 - (9/10)\$1000.00}{9/10 - 1} = \$20.00.$$

As a special case, if ever $x = 0$ formula (4) takes an indeterminate form which, however, is easily evaluated in the usual way to be (1), $k = (C - S)/n$, where periodic estimates are no longer needed and R_n is replaced by S. Likewise, if $k = 0$ formula (3) becomes $S = C(1 - x)^n$, where, again, periodic estimates are no longer needed and R_n is replaced by S. Solving this equation for x we obtain formula (2).

AN INTERESTING FOURTEENTH CENTURY TABLE.

By DAVID EUGENE SMITH, Columbia University.

There has recently come into the possession of the library of Columbia University an interesting mathematical roll written apparently in the south of England about the close of the fourteenth century. It is $3\frac{9}{16}$ inches wide and $38\frac{3}{16}$ inches long and consists of two strips of parchment sewed together, only the first part of the roll being shown in the facsimile. The style of the script, the spelling, and the forms of the numerals suggest as the approximate date the year 1400. This is rather early for a mathematical manuscript in the English language, although we have others of still earlier date. English manuscripts of a mathematical nature written before the fifteenth century usually concern the interests of the less scholarly class, and, since this particular specimen relates to farm measurement, it would have been of little service had it appeared in the Latin of the church schools.



The roll consists of a table showing the widths corresponding to various lengths of a rectangular piece of land containing an acre. As the facsimile shows, the first column gives the lengths, beginning with 1 rod, the caption reading: "This is the lenght of the acre of londe." The lengths are given for every rod from 1 to 160.